07-08 A1

.

$1.\lim_{x\to 0}\frac{\sin 3x}{x}f\left(\frac{\sin 2x}{x}\right)=3f(2)=9$

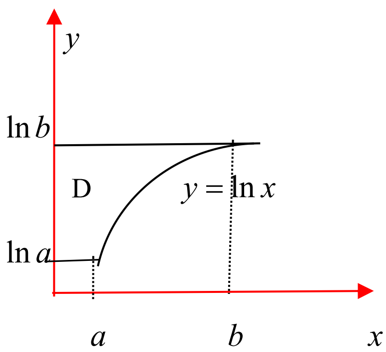
$2.K=\frac{|y''|}{\left[1+(y')^2\right]^{2/3}}=\frac{2a}{\left[1+(y')^2\right]^{2/3}}$ $|y'|$ $y'=2ax+b=0\Rightarrow x=-\frac{b}{2a}$

$3.\tan y=x+y,$ $\sec^2 ydy=dx+dy\Rightarrow dy=\cot^2 ydx$

$4.\cos\langle a,b\rangle=\frac{a\cdot b}{|a||b|}=\frac{3-2+2}{\sqrt{9+1+4}\sqrt{1+1+4}}=\frac{3}{2\sqrt{21}}$

$1.\lim_{x\to 0}(1+\cos x)^{\frac{3}{\cos x}}=2^3=8$

$2A=\int_{\ln a}^{\ln b}e^y dy=\int_0^a(\ln b-\ln a)dx+\int_a^b(\ln b-\ln x)dx$



$3.M=\int_{-\frac{\ }{2}}^{\frac{\ }{2}}\frac{\sin x}{1+x^2}\cos^4 xdx=0($ $)$

$.N=2\int_0^{\frac{\ }{2}}\cos^4 xdx>0, P=-2\int_0^{\frac{\ }{2}}\cos^4 x<0$

$M<N<P$

4. $x=x_0$ $U(x_0,\ )$, $f(x)=f(x_0)+\frac{f'''(x_0)}{3!}(x-x_0)^3$

$f'(x)=f'''(x_0)(x-x_0)^2,$ 0

$f'(x)=0\Rightarrow x=x_0$ $f'(x)<0$ $x=x_0$

$f''(x)=2f'''(x_0)(x-x_0)$ $(x_0,f(x_0))$

$1.\lim_{h\to 0}\frac{f(a-h)-f(a+2h)}{h}=\lim_{h\to 0}-\frac{f(a-h)-f(a)}{-h}-2\lim_{h\to 0}\frac{f(a+2h)-f(a)}{2h}=-f'(a)+2f'(a)=f'(a)$

$2\lim_{x\to\frac{\ }{2}}\left(x-\frac{\ }{2}\right)\cot 2x\overset{u=x-\frac{\ }{2}}{==}\lim_{u\to 0}\frac{u}{\tan 2u}=\frac{1}{2}$

$$1\int\frac{10^{2\arccos x}}{\sqrt{1-x^2}}dx=-\frac{1}{2}\int 10^{2\arccos x}d(2\arccos x)=-\frac{10^{2\arccos x}}{2\ln 10}+C$$

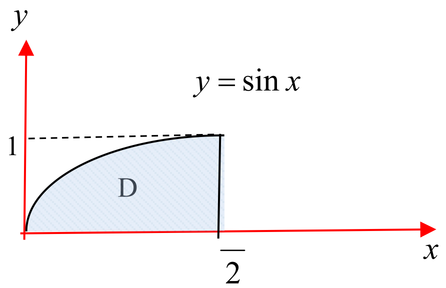
$$2.\int_0^2 f(x-1)dx\overset{x-1=u}{==}\int_{-1}^{1}f(u)du=\int_{-1}^{0}\frac{1}{1+e^x}dx+\int_0^1\frac{1}{1+x}dx=\ln 2+\int_{-1}^{0}\frac{1+e^x-e^x}{1+e^x}dx$$
$$=\ln 2-1+\left[\ln\left(1+e^x\right)\right]_{-1}^{0}=2\ln 2-1+\ln\left(1+e^{-1}\right)$$

$$3.\int_0^{\frac{}{2}}e^{2x}\sin 2xdx=\left[\frac{1}{2}e^{2x}\sin 2x\right]_0^{\frac{}{2}}-\int_0^{\frac{}{2}}e^{2x}\cos 2xdx=-\frac{1}{2}\left[e^{2x}\cos 2x\right]_0^{\frac{}{2}}-\int e^{2x}\sin 2xdx$$
$$=\frac{1}{2}e\quad+\frac{1}{2}-\int_0^{\frac{}{2}}e^{2x}\sin 2xdx\Rightarrow\int_0^{\frac{}{2}}e^{2x}\sin 2xdx=\frac{1}{4}\left(e\quad+1\right)$$

$$1.x_t'\big|_{t=0}=2e^t=2,y_t'\big|_{t=0}=-e^{-t}=-1,x(0)=2,y(0)=1\quad K\qquad=\frac{2}{-1}=-2.K\qquad=\frac{1}{2}$$
$$y-1=-2(x-2)\Rightarrow y=-2x+5$$
$$y-1=\frac{1}{2}(x-2)\Rightarrow y=\frac{1}{2}x$$

$$2.V=\frac{1}{3}\quad r^2h=\frac{1}{3}\quad h\left(20^2-h^2\right)\quad V'=\frac{400}{3}\quad-\quad h^2=0\Rightarrow h=\frac{20}{\sqrt{3}}$$
$$h=\frac{20}{\sqrt{3}}$$

$$3.V=2\quad\int_0^{\frac{}{2}}xydx=2\quad\int_0^{\frac{}{2}}x\sin xdx=2\quad\left[-x\cos x+\sin x\right]_0^{\frac{}{2}}=2$$



$$A,\quad\lim_{n\to\infty}x_n=A$$
$$x_n=\sqrt{2+x_n}\qquad,\quad A=\sqrt{2+A}\Rightarrow A=2$$
$$\left|x_n-2\right|=\left|\sqrt{2+x_{n-1}}-2\right|$$

$$=\left|\frac{x_{n-1}-2}{\sqrt{2+x_{n-1}}+2}\right|<\frac{1}{4}\left|x_{n-1}-2\right|<\frac{1}{4^2}\left|x_{n-2}-2\right|.....<\frac{1}{4^{n-1}}\left|x_1-2\right|\to 0(n\to 0)$$

$$F'(x)=f(x)+\frac{1}{f(x)}\geq 2>0 \qquad F(x)$$

$$(2)F(b)=\int_a^b f(t)dt>0, F(a)=\int_b^a \frac{1}{f(t)}dt<0$$

$F(b)F(a)<0,\quad F(x)$,

$$e^x\left(f(x)+f'(x)\right)=0\Rightarrow\left[e^x f(x)\right]'=0\Rightarrow e^x f(x)=C$$

$f(0)=1\Rightarrow C=1.\quad f(x)=e^{-x}$

08-09 A1

$$1.\lim_{x\to 0}\left(x\sin\frac{1}{x}-\frac{1}{x}\sin x\right)=0-1=-1$$

$$2.y'=f'(\ln x)\frac{1}{x}\Rightarrow dy=\frac{f'(\ln x)}{x}dx$$

$$3.\int\left(e^x-3\cos x\right)dx=e^x-3\sin x+C$$

$$4.\int_{-a}^{a}\left(x^3+\sin^3 x\right)dx=0(\qquad)$$

$$1.\lim_{x\to 0}\frac{f(x)-f(0)}{x}=f'(0)=2\Rightarrow$$

$$2.y(-x)=-y(x)$$

$$3.\overrightarrow{NM}=(3,-4,5).\overrightarrow{NP}=(-1,-2,2).\cos\angle MNP=\frac{\overrightarrow{NM}\cdot\overrightarrow{NP}}{\left|\overrightarrow{NM}\right|\left|\overrightarrow{NP}\right|}=\frac{-3+8+10}{\sqrt{50\times 9}}=\frac{\sqrt{2}}{2}\therefore\angle MNP=\frac{}{4}$$

$4.f'''(x)>0, f''(0)=0\Rightarrow f''(x)>0\Rightarrow f'(x)$

$f(x)$, $f'(0)<\frac{f(1)-f(0)}{1-0}=f'(\ )<f'(1).\ \in(0,1)$

5. $\int_0^1 f(x)dx=A\Rightarrow f(x)=x+2A$ $(0,1)$, $A=\frac{1}{2}+2A\Rightarrow A=-\frac{1}{2}$

$$1.\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}=\lim_{x\to 0}\frac{\sin x(1-\cos x)}{\cos x\sin^3 x}=\lim_{x\to 0}\frac{x\left(\frac{1}{2}x^2\right)}{x^3}=\frac{1}{2}$$

2.

$$3.\lim_{x\to0^+}(\sin x)^x=\lim_{x\to0^+}e^{x\ln\sin x}=\lim_{x\to0^+}e^{\frac{\ln\sin x}{\frac{1}{x}}}\overset{\frac{\infty}{\infty}.}{===}\lim_{x\to0^+}e^{-\frac{x^2}{\tan x}}=e^0=1$$

$$1.x'=2t,y'=-\sin t.\frac{dy}{dx}=\frac{y'}{x'}=-\frac{\sin t}{2t}.\frac{d^2y}{dx^2}=\frac{d\left(\frac{dy}{dx}\right)}{dt}\frac{dt}{dx}=\frac{-\cos t2t+2\sin t}{4t^2 2t}=\frac{\sin t-t\cos t}{4t^3}$$

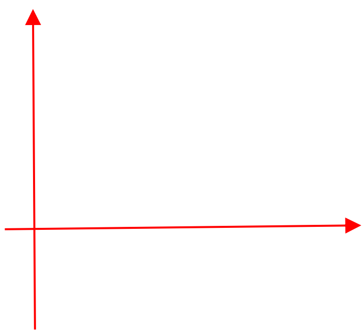
$$2.\int\frac{xe^{x^2}}{1-2e^{x^2}}dx=\frac{1}{2}\int\frac{d\left(e^{x^2}\right)}{1-2e^{x^2}}=-\frac{1}{4}\ln\left|1-2e^{x^2}\right|+C$$

$$3.\int_1^4\frac{\ln x}{\sqrt{x}}dx=2\sqrt{x}\ln x\Big|_1^4-\int_1^4\frac{2}{\sqrt{x}}dx=8\ln2-4\sqrt{x}\Big|_1^4=8\ln2-4$$

$$F(x)=\sin x+\tan x-2x.F'(x)=\cos x+\sec^2 x-2=\cos x+\tan^2 x-1$$
$$G(x)=\cos x-1+\tan^2 x.G'(x)=-\sin x+2\tan x\sec^2 x=\sin x\left(2\sec^3 x-1\right)$$
$$\because 0<\cos x<1.x\in\left(0,\frac{}{2}\right)\therefore\sec x=\frac{1}{\cos x}>1.\quad 2\sec^3 x-1>0$$
$$\Rightarrow G'(x)>0.G(0)=0.\Rightarrow G(x)>0,\quad f'(x)>0.f(0)=0.\therefore f(x)>0$$
$$\Rightarrow\sin x+\tan x>2x$$

$$\Phi(x)=\int^x f(x)dx=\int^{-\infty}f(x)dx+\int_{-\infty}^x f(x)dx=-1+\int_{-\infty}^x f(x)dx$$
$$x<0.\Phi(x)=-1.0<x<\quad,\Phi(x)=-1+\int_{-\infty}^x f(x)dx=-1+\int_0^x\frac{1}{2}\sin xdx=-1+\frac{1}{2}(-\cos x)\Big|_0^x=-\frac{1}{2}-\frac{1}{2}\cos x$$
$$x>\quad.\Phi(x)=-1+\int_{-\infty}^x f(x)dx=-1+\int_{-\infty}^0 0dx+\int_0\frac{1}{2}\sin xdx+\int^x 0dx=0$$
$$\therefore\Phi(x)=\begin{cases}-1 & ,x<0\\ -\frac{1}{2}-\frac{1}{2}\cos x\le x\le & \\ 0 & ,x>\end{cases}$$

$$f(x)=\int_a^x f(x)dx\Rightarrow f(a)=\int_a^a f(x)dx=0\qquad .f'(x)=f(x)\Rightarrow f(x)=Ce^x$$
$$x=a.f(a)=Ce^a=0\Rightarrow C=0\quad\therefore f(x)=\begin{cases}0,x=a\\ Ce^x,x\neq a\end{cases}$$



$$S=\int_0^1 ax+bx^2dx=\frac{a}{2}+\frac{b}{3}=\frac{4}{9}\Rightarrow b=\frac{4}{3}-\frac{3}{2}a$$

$$V=\quad\int_0^1 y^2dx=\quad\left[\int_0^1\left(a^2x^2+\left(\frac{4}{3}-\frac{3}{2}a\right)^2x^4+2a\left(\frac{4}{3}-\frac{3}{2}a\right)x^3\right)\right]dx=\quad\left(\frac{1}{3}a^2+\frac{1}{5}\left(\frac{4}{3}-\frac{3}{2}a\right)^2+\frac{a}{2}\left(\frac{4}{3}-\frac{3}{2}a\right)\right)$$

$$V'=\quad\left(\frac{2}{3}a-\frac{3}{5}\left(\frac{4}{3}-\frac{3}{2}a\right)+\frac{1}{2}\left(\frac{4}{3}-\frac{3}{2}a\right)-\frac{3}{4}a\right)=\quad\left(\frac{2}{5}a-\frac{2}{15}\right)=0\Rightarrow a=\frac{1}{3}.b=\frac{4}{3}-\frac{3}{2}\frac{1}{3}=\frac{5}{6}$$

09-10 A1

$$1.\lim_{x\to0}\frac{1-\cos x}{x^2}=1/2(\qquad)$$

$$2.\lim_{x\to0}x^2\sin\frac{1}{x}=0=f(0),$$

$$\lim_{x\to0}f'(0)=\lim_{x\to0}\frac{x^2\sin\frac{1}{x}-0}{x}=\lim_{x\to0}x\sin\frac{1}{x}=0=f'(0)$$

$$3.\lim_{x\to1^-}f(x)=\lim_{x\to1^-}\frac{x^2-1}{x-1}=\lim_{x\to1^-}x+1=2$$
$$\lim_{x\to1^+}f(x)=2x=2,$$

4.C

$$1.\lim_{x\to+\infty}\left(1-\frac{1}{x}\right)^{3x}=\lim_{x\to+\infty}\left(1-\frac{1}{x}\right)^{-x\times(-3)}=e^{-3}$$

$$2.\lim_{x\to0}\frac{e^{3x\sin x}-1}{\tan x^2}=\lim_{x\to0}\frac{2x\sin x}{\tan x^2}=2$$

$$3.k=\lim_{x\to\infty}\frac{y}{x}=\lim_{x\to\infty}\frac{x}{3x+1}=\frac{1}{3}\qquad b=\lim_{x\to\infty}(y-kx)=\lim_{x\to\infty}\frac{-3x}{9x+3}=-\frac{1}{3}$$

$$y=\frac{1}{3}(x-1)$$

( ) ( )

$8.\overline{y}=\frac{1}{}\int_0 \sin x dx=\frac{2}{}$

$9.\int_{-1}^{1}(|x|+\sin x)x^2dx=2\int_0^1 x^3dx=\frac{1}{2}$

10. $\int_0^1 f(x)dx=A.f(x)=x+2A$ $[0,1]$ $A=\frac{1}{2}+2A\Rightarrow A=-\frac{1}{2}$

$1.x'=2t+1, y'=\cos t \quad \frac{dy}{dx}=\frac{y'}{x'}=\frac{\cos t}{2t+1},\frac{d^2y}{dx^2}=\frac{d\left(\frac{dy}{dx}\right)}{dt}\frac{dt}{dx}=\frac{-\sin t(2t+1)+2\cos t}{(2t+1)^3}$

$2.\sin(xy)+\ln(y-x)=x$

$\cos(xy)(xdy+ydx)+\frac{dy-dx}{y-x}=dx,(0,1)$

$dx+dy-dx=dx\Rightarrow\frac{dy}{dx}=1$ $y=x+1$

$1.\lim_{x\to0}\frac{e^x-(1+2x)^{\frac{1}{2}}}{\ln(1+x^2)}=\lim_{x\to0}\frac{e^x-e^{\frac{1}{2}\ln(1+2x)}}{x^2}=\lim_{x\to0}\frac{e^x\left[1-e^{\frac{1}{2}\ln(1+2x)-x}\right]}{x^2}=\lim_{x\to0}\frac{-\frac{1}{2}\ln(1+2x)-x}{x^2}$

$\overset{\frac{0}{0}}{=====}\lim_{x\to0}\frac{-\frac{1}{1+2x}-1}{2x}=-\frac{1}{4}$

$2.n\frac{n}{n+n^2}<A<n\frac{1}{1+n^2}$ $\mathrm{A}=1$

$<x<$ $\Phi(x)=\int_0^x f(x)dx=\frac{}{}x^2$

$\leq x<$ $\Phi(x)=\int_0^x f(x)dx=\int_0^3 f(x)dx+\int_3^x f(x)dx=\frac{3}{4}+\int_3^x 2-\frac{x}{2}dx=2x-3-\frac{1}{4}x^2$

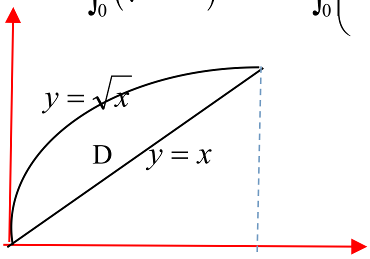
$$\frac{dy}{dx}\sin x = y\ln y \Rightarrow \frac{dy}{y\ln y} = \frac{dx}{\sin x} \Rightarrow \ln\ln y = \ln\left|\csc x - \cot x\right| + \ln c$$

$$\Rightarrow \ln y = c(\csc x - \cot x) \quad y(\ \ /2) = e, \quad c = 1 \therefore y = \csc x - \cot x$$

$$2.(x^2-1)y' + 2xy = \cos x, \quad \left[(x^2-1)y\right] = \cos x \Rightarrow (x^2-1)y = \sin x + C$$

$$A = \int_0^1 (\sqrt{x} - x)dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^1 = \frac{1}{6}$$

$$V = \quad \int_0^1 (\sqrt{x} - x)^2 dx = \quad \int_0^1 \left(x + x^2 - x^{\frac{3}{2}}\right)dx = \frac{13}{25}$$



$$(2).\quad F(x) = f(x) - \sin x \quad F(0) = f(0) - 0 = 0, F(\ \ /2) = f(\ \ /2) - 1 = 0$$

$$, \qquad \subset \left(0, \frac{\ }{2}\right)$$

$$F'(x) = f'(x) - \cos x = 0 \Rightarrow f'(x) = \cos x$$

10-11 A1

$$1.\lim_{x\to 0}\frac{\sqrt{1+x} - \sqrt{1-x}}{x} = \lim_{x\to 0}\frac{(1+x)-(1-x)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)} = \frac{2x}{2x} = 1$$

2.D

$$3.\quad F(x) = e^x - x - 2 \quad F'(x) = e^x - 1 > 0 \Rightarrow x > 0 \quad F(0) = -1, F(1) = e - 2 > 0$$

$(0,1)$

4. $f(x)$, $F(x) - G(x) = C$

$$1.\lim_{x\to 1^-} e^{\frac{1}{x-1}} = e^{-\infty} = 0$$

$$2.\lim_{x\to 0}\frac{\sin 3x}{x(1+\cos x)} = \lim_{x\to 0}\frac{3x}{2x} = \frac{3}{2}$$

3. , $e^y dy + y dx + x dy = 0 \Rightarrow dy = -\frac{y}{e^y + x} dx$

$$4. \lim_{\to 0} \quad = \lim_{\to 0} \quad ^{-\frac{1}{2}} = \lim_{\to 0} \frac{^{\frac{1}{2}}}{1} \overset{\frac{\infty}{\infty}}{===} \lim_{\to 0} \frac{-\frac{2}{3} e^{\frac{1}{2}}}{-\frac{1}{x^2}} = \lim_{\to 0} \frac{2e^{\frac{1}{2}}}{x} = \infty$$

$x = 0$

$1(1).\Rightarrow dy=\frac{\arctan x}{1+x^2}dx\Rightarrow y=\frac{1}{2}(\arctan x)^2+C\quad y|_{x=0}=1,\quad C=1\therefore 2y=(\arctan x)^2+2$

(2) , $\frac{dy}{dx}=\frac{1}{\sqrt{9-x^2}}\Rightarrow y=\arcsin\frac{x}{3}+C$

$(0,1)\Rightarrow C=1.$ $\quad y=\arcsin\frac{x}{3}+1$

$2.\Rightarrow\frac{xy'-y}{x^2}=\cos x.\quad\left(\frac{y}{x}\right)'=\cos x\Rightarrow\frac{y}{x}=\sin x+C$

$-1<x<0.\Phi(x)=\int_{-1}^{x}2x+\frac{3}{2}x^2=x^2+\frac{1}{2}x^3-\frac{1}{2}$

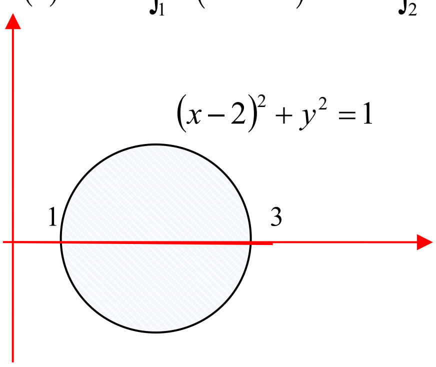
$0\le x<1.\Phi(x)=\int_{-1}^{0}2x+\frac{3}{2}x^2dx+\int_0^x\frac{1}{1+e^x}dx=\frac{1}{2}+\int_0^x\frac{1+e^x-e^x}{1+e^x}dx=\frac{1}{2}+x-\ln(1+e^x)$

$$\therefore\Phi(x)=\begin{cases}x^2+\frac{1}{2}x^3-\frac{1}{2},-1<x<0\\ \frac{1}{2}+x-\ln(1+e^x),0\le x<1\end{cases}$$

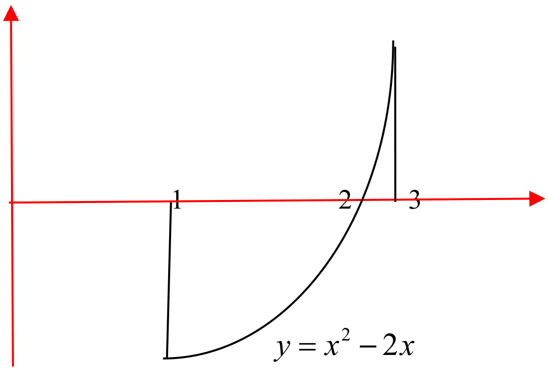
$(1)V=4\ \int_1^3 x\sqrt{1-(x-2)^2}dx\overset{x-2=\sin t}{==}4\ \int_{-\frac{}{2}}^{\frac{}{2}}(2+\sin t)\cos^2 t\,dt=4\ \int_0^{\frac{}{2}}2\cos^2 t\,dt=2\ ^2$

$(2)A=\int_0^2|y|dx=\int_0^2 2x-x^2dx=\frac{4}{3}$

$(2)V=2\ \int_1^2 x(2x-x^2)dx+2\ \int_2^3 x(x^2-2x)dx=9$



(1)



(2)

$1(1)\int_0^2 f(x)dx=\int_0^1 f(x)-f(0)dx+\int_1^2 f(x)-f(2)dx$

$\therefore\left|\int_0^2 f(x)dx\right|=\left|\int_0^1 f(x)-f(0)dx+\int_1^2 f(x)-f(2)dx\right|=\left|\int_0^1 f'(\ _1)xdx+\int_1^2 f'(\ _2)(x-2)dx\right|$

$$\le\left|\int_0^1 f(\ _1)xdx\right|+\left|\int_1^2 f(\ _2)(2-x)dx\right|\le M\int_0^1 xdx+M\int_1^2(2-x)dx=\frac{1}{2}M+\frac{1}{2}M=M(\ _1\in(0,1),\ \ _2\in(1,2))$$

$$\Rightarrow\left|\int_0^2 f(x)dx\right|\le M$$

(2) (p,q) , , $_1$

$f'(\ _1)=0,(\ _1\in(p,q))$

$,(q,r)$, $_2$, $f'(\ _2)=0$

$(\ _1,\ _2)$ $\in(\ _1,\ _2)\subset(a,b)$

$f''(\ )=0$

11-12 A1

1. --D

2. $\lim_{x\ \ 0}\frac{f^2(x\ \ \ x)\ \ f^2(x)}{}$ $^2(\ )\ '$ $2\ (\ )\ '(\ )$

7.0(, .)

$8. dA=\frac{1}{2}\quad{}^{2}(\quad)d \qquad V=\int_c^d \quad {}^{2}(y)dy$

$$1.\lim_{x\to\infty}\left(\frac{2x+1}{2x-1}\right)^{2x}=\lim_{x\to\infty}\left(1+\frac{2}{2x-1}\right)^{2x}=\lim_{x\to\infty}\left(1+\frac{2}{2x-1}\right)^{\frac{2x-1}{2}\cdot\frac{4x}{(2x-1)}}=e^{\frac{1}{2}}$$

$$2. n\frac{n}{n^2+n}<A<n\frac{n}{n^2+}$$

, $A=1$

$$3. x'=1-\frac{1}{1+t}=\frac{t}{1+t}, y'=3t^2+2t \quad \frac{dy}{dx}=\frac{y'}{x'}=\frac{(3t^2+2t)(1+t)}{t}=(3t+2)(1+t)=3t^2+5t+2$$

$$\frac{d^2y}{dx^2}=\frac{d\left(\frac{dy}{dx}\right)}{dt}\frac{dt}{dx}=\frac{(6t+5)(1+t)}{t}$$

$$F(x)=2x-\ln\frac{1+x}{1-x}$$

$$F'(x)=2-\frac{1}{1+x}-\frac{1}{1-x}=2-\frac{2}{1-x^2}=\frac{-x^2}{1-x^2}<0. \quad (0,1)$$

$$F(0)=0. \quad F(0)<0\Rightarrow 2x-\ln\frac{1+x}{1-x}<0\Rightarrow e^{2x}<\frac{1+x}{1-x}$$

.

$$1.\int\ln\sin x\csc^2 x dx=-\ln\sin x\cot x+\int\cot^2 x dx=-\ln\sin x\cot x+\int(\csc^2 x-1)dx$$

$$=-\ln\sin x\cot x-\cot x-x+C$$

$$\int xe^{-2x}dx=-\frac{1}{2}xe^{-2x}+\frac{1}{2}\int e^{-2x}dx=-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}+C$$

$$2.\int\frac{x-2}{x^2+2x+3}dx=\frac{1}{2}\int\frac{2x+2-6}{x^2+2x+3}dx=\frac{1}{2}\int\frac{d(x^2+2x+3)}{x^2+2x+3}-3\int\frac{1}{(x+1)^2+2}dx$$

$$=\frac{1}{2}\ln(x^2+2x+3)-\frac{3}{\sqrt{2}}\arctan\frac{x+1}{2}+C$$

$3.\quad \int_0^1 f(x)dx=A \qquad (0,1) \qquad , \quad A=\int_0^1\frac{1}{1+x^2}dx+A\int_0^1\frac{1}{\sqrt{1-x^2}}dx$

$$\Rightarrow A=\frac{}{4}+\frac{}{2}A\Rightarrow A=\frac{}{4-2}$$

$$-1\le x\le 3, F(x)=\int_{-\infty}^{-1}f(t)dt+\int_{-1}^{x}f(t)dt=\int_{-1}^{x}(1-t^2)dt=\left(t-\frac{1}{3}t^3\right)_{-1}^{x}=x-\frac{1}{3}x^3+\frac{2}{3}$$

$$x \quad 3, F\left(x\right)=\int_{-\infty}^{-1} f\left(t\right)dt+\int_{-1}^{3} f\left(t\right)dt+\int_{3}^{x} f\left(t\right)dt=\left(t-\frac{1}{3}t^{3}\right)_{-1}^{3}=-\frac{16}{3}$$

$$\therefore F\left(x\right)=\begin{cases} 0, x \quad -1 \\ x-\frac{1}{3}x^{3}+\frac{2}{3}, -1\le x\le 3 \\ -\frac{16}{3}, x \quad 3 \end{cases} \qquad \lim_{x\to 3^{-}} F\left(x\right)=\left(3-\frac{1}{3}3^{3}+\frac{2}{3}\right)=-\frac{16}{3}=\lim_{x\to 3^{+}} F\left(x\right)=F\left(3\right)$$

$f\left(3^{+}\right)=0. f\left(3^{-}\right)=-8 \quad \therefore \mathrm{F}\left(x\right) \quad x=3$

.

$7.4x^{2}+y^{2}=4, 8x+2yy'=0 \Rightarrow y'=-\frac{4x}{y} \therefore$ $\qquad Y-y=-\frac{4x}{y}\left(X-x\right)$

$X=0, \quad Y=\frac{4}{y}. Y=0. \quad X=\frac{1}{x}$ ((

$S \quad \frac{1}{2}XY \quad \frac{1}{4}2 \quad \frac{2}{xy} \quad \frac{}{2} \quad \frac{2}{x\sqrt{4 \quad 4x^{2}}} \quad \frac{}{2} \quad \frac{1}{x\sqrt{1 \quad x^{2}}} \quad \frac{}{2}$

$.f(c) < f(a) = f(b)$, (c,b) .

$f'(\ ) = \dfrac{f(b) - f(c)}{b - c} > 0$

$- \quad \in (a,b) \quad f'(\ ) > 0$

12-13 A1

1. $\lim_{x\to 0^+} \frac{1}{x} = +\infty$,$\lim_{x\to 0^-} \frac{1}{x} = -\infty$ $\lim_{x\to 0} \frac{1}{x}$ $---D$

2. $\lim_{x\to\infty} y = \lim_{x\to\infty} \dfrac{1+e^{-2x^2}}{1-e^{-x^2}} = 1 --$ $y = 1$

$\lim_{x\to 0} \dfrac{1+e^{-x^2}}{1-e^{-x^2}} = \dfrac{2}{0} = \infty ---$ $x = 0$

3. $---D$

4. $f'(\sin^2 x) = 1 - \cos^2 x \Rightarrow f'(x) = 1 - x$

$\Rightarrow f(x) = x - \frac{1}{2}x^2 + C$

1. $\lim_{x\to\infty}\left(\dfrac{x-1}{x}\right)^{2x} = \lim_{x\to\infty}\left(1 - \dfrac{1}{x}\right)^{-x\times(-2)} = e^{-2}$

2. $y' = \dfrac{1}{x}.y'' = -\dfrac{1}{x^2}.y''' = (-1)^2 \dfrac{2}{x^3}....y^{(n)} = (-1)^{n-1}\dfrac{(n-1)!}{x^n}$

3. $, xdy + ydx = e^{x+y}(dx + dy)$

$\Rightarrow xdy + ydx = xy(dx + dy) \Rightarrow dy = \dfrac{xy - y}{x - xy}dx$

4. $y' = e^{-x} - xe^{-x}.y'' = -e^{-x} + xe^{-x} - e^{-x} = (2 - x)e^{-x} = 0 \Rightarrow x = 2$

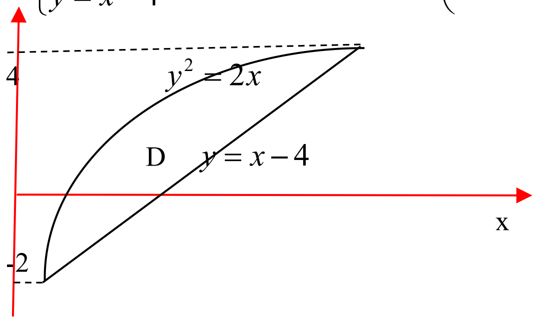
$x = 0$ $.y''$. $(2, 2e^{-2})$

5. $\dfrac{d}{dx}\int_0^{x^2} \ln(1+t)dt = 2x\ln(1+x^2)$

6. $\int_e^{+\infty} \dfrac{dx}{x\ln^2 x} = \int_e^{+\infty} \dfrac{d(\ln x)}{\ln^2 x} = -\dfrac{1}{\ln x}\Big|_e^{+\infty} = 1$

7. $(x_0) = \lim_{x\to x_0} \dfrac{m(x) - m(x_0)}{x - x_0} = [m(x)]'\Big|_{x=x_0} = m'(x_0)$

$$8.\begin{cases} y^2 = 2x \\ y = x-4 \end{cases} \Rightarrow (2,-2).(8,4) \therefore dA = \left(y + 4 - \frac{y^2}{2} \right) dy$$



$$1.\lim_{x\to 0} \frac{\sin 4x + x^2 \sin \frac{1}{x}}{(1+\cos x)x} = \lim_{x\to 0} \frac{\sin 4x}{2x} = 2$$

$$2.\lim_{x\to 0} \frac{x - \arcsin x}{x^3} \overset{\arcsin x = t}{==} \lim_{t\to 0} \frac{\sin t - t}{\sin^3 t} \overset{\frac{0}{0}.}{====} \lim_{x\to 0} \frac{\cos t - 1}{3t^2} = \lim_{x\to 0} \frac{-\frac{1}{2}t^2}{3t^2} = -\frac{1}{6}$$

$$3.y = e^{x^2 \ln \frac{x+2}{1-x}}.y' = e^{x^2 \ln \frac{x+2}{1-x}} \left(2x \ln \frac{x+2}{1-x} + x^2 \left(\frac{1}{x+2} + \frac{1}{1-x} \right) \right) = \left(\frac{2+x}{1-x} \right)^{x^2} \left(2x \ln \frac{2+x}{1-x} + \frac{3x^2}{(2+x)(1-x)} \right)$$

$$4.x' = 2t - \cos t + t \sin t + \cos t = t(2+\sin t).y' = 2 + \sin t$$

$$\frac{dy}{dx} = \frac{y'}{x'} = \frac{1}{t}, \frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dt} \frac{dt}{dx} = -\frac{1}{2t^2(2+\sin t)}$$

$$1.\int \frac{x+1}{x\sqrt{x-2}} dx = \int \frac{1}{\sqrt{x-2}} dx + \int \frac{dx}{x\sqrt{x-2}} = 2\sqrt{x-2} + \int \frac{dx}{(x-2+2)\sqrt{x-2}}$$

$$= 2\sqrt{x-2} + 2\int \frac{d\left(\sqrt{x-2}\right)}{2 + \left(\sqrt{x-2}\right)^2} = 2\sqrt{x-2} + \sqrt{2} \arctan \frac{\sqrt{x-2}}{2} + C$$

$$2\int_0^{\frac{1}{2}} x \arcsin x dx \xrightarrow[dx=\text{cosudu}]{\arcsin x = u, x=0, u=0, x=\frac{1}{2}, u=\frac{}{6}} = \int_0^{\frac{}{6}} u \sin u \cos u du = \frac{1}{2} \int_0^{\frac{}{6}} u \sin 2u du$$

$$= \left(-\frac{1}{4} u \cos 2u + \frac{1}{8} \sin 2\text{u} \right)_0^{\frac{}{6}} = \frac{3\sqrt{3} -}{48}$$

$$0 < x \le 1.\Phi(x) = \int_0^x f(t)dt = \int_0^x e^t dt = e^x - 1$$

$$x > 1.\Phi(x) = \int_0^x f(t)dt = \int_0^1 e^x dx + \int_1^x \frac{1}{x} dx = e - 1 + \ln x$$

$$\therefore \Phi(x) = \begin{cases} e - 1 + \ln x, x > 1 \\ e^x - 1.0 < x \le 1 \end{cases}$$

$\lim\limits_{x\to1^+}\Phi(x)=e-1.\lim\limits_{x\to1^-}\Phi(x)=e-1=\Phi(1)$

$\lim\limits_{x\to1^+}f(x)=1.\lim\limits_{x\to1^-}f(x)=1$

$\therefore\Phi(x)\quad x=1$

$$V=\int_0^2 y^2(t)dx=8\int_0^2(1-\cos t)^3dt=8\int_0^2\left(2\sin^2\frac{t}{2}\right)^3dt=2^6\int_0^2\sin^6\frac{t}{2}dt$$

$$\overset{2u=t}{=\!=\!=}2^6\int_0\sin^6 udu=2^7\int_0^{\frac{}{2}}\sin^6 udu=\frac{5}{6}\frac{3}{4}\frac{1}{2}\frac{}{2}2^7\quad=20\quad^2$$

.

$F(t)=f(t)-t$

$F(0)=f(0)>0.F(1)=f(1)-1<0$

$F(0)F(1)<0.\quad F(t)\quad(0,1)\qquad x.\quad F(x)=0\Rightarrow f(x)=x$

$F'(t)=f'(t)-1\neq0$

$f(x)=x$

$,\frac{1}{b-a}\int_a^b f(x)dx=f(c)=f(b).c\in(a,b)$

$(b,c)\qquad ,\qquad \in(c,b)\subset(a,b)$

$f'(\quad)=0$

13-14 A1

$1.\lim\limits_{x\to0}\sin x\sin\frac{1}{x}=0.$

$2.---D$

$3.p=0.\int_0^1\frac{1}{x}dx=-\infty$

$p=2.\int_0^1\frac{dx}{x^{1-p}}dx=\int_0^1 xdx=\frac{1}{2}\qquad ---D$

$4.0(\qquad , \qquad)$

$$1.\lim_{x\to0}\frac{\cos x-1}{\left(1+x^2\right)^{\frac13}-1}=\lim_{x\to0}\frac{-\frac12x^2}{\frac13x^2}=-\frac32$$

$$2.y'=\frac1x=2\Rightarrow x=1/2\Rightarrow\quad(1/2,-\ln2)\qquad\qquad y=2x-3$$

3. , $\sin xdy+y\cos xdx+\sin(x-y)(dx-dy)=0$

$$\Rightarrow dy=\frac{y\cos x+\sin(x-y)}{\sin(x-y)-\sin x}dx$$

$$4.\int e^{1+\sin^2x}\sin2xdx=\int e^{1+\sin^2x}d\left(1+\sin^2x\right)=e^{1+\sin^2x}+C$$

$$5.F(x)=\int_0^xf(t)dt=\int_0^3\left(1-x^2\right)dx+\int_3^x0dx=x-\frac13x^3\Big|_0^3=-6$$

$$6.V=\int_a^b(f(x)-g(x))^2dx$$

$$1.\lim_{x\to+\infty}\left(1-\frac1{x^2}\right)^{3x}=\lim_{x\to+\infty}e^{3x\ln\left(1-\frac1{x^2}\right)}=\lim_{x\to+\infty}e^{-3x\times\frac1{x^2}}=e^0=1$$

$$2.x'=-e^t\sin t+e^t\cos t,y'=e^t(\sin t+\cos t)\quad\frac{dy}{dx}=\frac{y'}{x'}=\frac{\sin t+\cos t}{\cos t-\sin t}$$

$$\frac{d^2y}{dx^2}=\frac{d\left(\frac{dy}{dx}\right)}{dt}\frac{dt}{dx}=\frac{(\sin t-\cos t)^2+(\sin t+\cos t)^2}{(\cos t-\sin t)^2}\frac{1}{e^t(\cos t-\sin t)}=\frac{2}{e^t(\cos t-\sin t)^3}$$

$$3.\lim_{x\to+\infty}\frac{\int_1^x\left[t^2\left(e^{\frac1t}-1\right)-t\right]}{x}\overset{\frac{\infty}{\infty}.}{===}\lim_{x\to+\infty}x^2\left(e^{\frac1x}-1\right)-x\overset{u=\frac1x}{===}\lim_{u\to0^+}\frac{e^u-1-u}{u^2}\overset{\frac00.}{===}\lim_{u\to0^+}\frac{e^u-1}{2u}=\frac12$$

$$1.\int_0^{\frac{\ ^2}{4}}\sin\sqrt{x}dx\overset{\sqrt{x}=t}{===}2\int_0^{\frac{}{2}}t\sin tdt=-2t\cos t\Big|_0^{\frac{}{2}}+2\int_0^{\frac{}{2}}\sin tdt=\quad-2$$

$$\ln f(x)=3\ln(1+x)-2\ln(x-1)$$

$$\Rightarrow\frac{f'(x)}{f(x)}=\frac3{1+x}-\frac2{x-1}=\frac{x-5}{x^2-1}\Rightarrow f'(x)=\frac{(x+1)^2}{(x-1)^2}\frac{x-5}{x-1}<0\Rightarrow1<x<5$$

$$f'(x)>0\Rightarrow x>5\quad x<1.f\quad(x)=f(5)$$

$$\ln f'(x)=2\ln(1+x)+\ln(x-5)-3\ln(x-1)$$

$$\frac{f''(x)}{f'(x)}=\frac2{1+x}+\frac1{x-5}-\frac3{x-1}=\frac{24}{(x+1)(x-5)(x-1)}\Rightarrow f''(x)=\frac{24}{(x+1)(x-5)(x-1)}\frac{(x+1)^2(x-5)}{(x-1)^3}$$

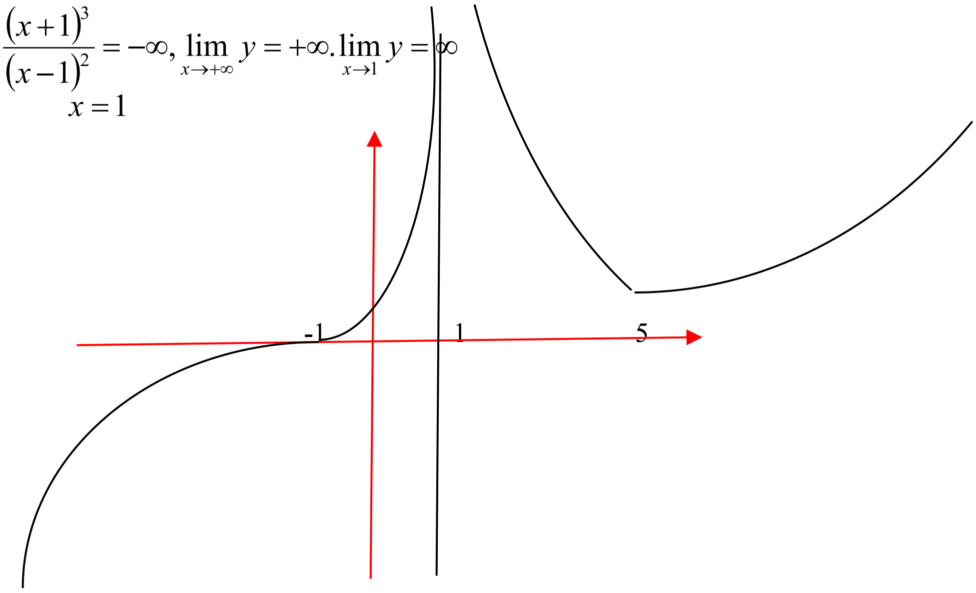
$$=\frac{24(x+1)}{(x-1)^4}>0\Rightarrow x>-1.f(-1)$$

x	(- ,-1)	-1	(-1,1)	1	(1,5)	5	(5,+)

$$k=\lim_{x\to\infty}\frac{y}{x}=\lim_{x\to\infty}\frac{(x+1)^3}{(x-1)^2x}=1.b=\lim_{x\to\infty}(y-kx)=\lim_{x\to\infty}\frac{(x+1)^3-x(x-1)^2}{(x-1)^2}$$
$$=\lim_{x\to\infty}\frac{x^3+3x+3x^2+1-x^3+2x^2-x}{(x-1)^2}=5$$

$\therefore$ $y=x+5$. $f(-1)=0.f(5)=\frac{6^3}{4^2}$

$$\lim_{x\to-\infty}y=\lim_{x\to\infty}\frac{(x+1)^3}{(x-1)^2}=-\infty,\ \lim_{x\to+\infty}y=+\infty.\lim_{x\to1}y=\infty$$
$x=1$



$$x'=-\sin t+\sin t+t\cos t=t\cos t,\ y'=\cos t-\cos t+t\sin t$$
$$s=\int_0^2 ds=\int_0^2\sqrt{(x')^2+(y')^2}dt=\int_0^2 tdt=2\quad{}^2$$

$$2.\int\frac{2x-3}{x^2-2x+5}dx=\int\frac{2x-2-1}{x^2-2x+5}dx=\int\frac{d(x^2-2x+5)}{x^2-2x+5}-\int\frac{dx}{(x-1)^2+4}$$
$$=\ln(x^2-2x+5)+\frac{1}{2}\arctan\frac{x-1}{2}+C$$

$f(x)$ $[c,d]$. M, m

$m\le f(c)\le M\Rightarrow\ m\le\ f(c)\le\ M$

$m \le f(d) \le M \Rightarrow \quad m \le \quad f(c) \le \quad M$

$, \quad m \le \quad f(c) + \quad f(d) \le M$

$, \qquad \in [c, d]$

$f(c) + \quad f(d) = f(\quad)$

$$F(x) = \frac{1}{a}\int_0^a f(x)dx - \int_0^1 f(x)dx = \frac{1}{a}\int_0^a f(x)dx - \int_0^a f(x)dx - \int_a^1 f(x)dx$$

$$\longrightarrow F(x) = f(\ {}_1) - af(\ {}_1) - (1-a) f(\ {}_2) \quad (\ {}_1 \subset (0,a), \ {}_2 \subset (a,1))$$

$$F(x) = f(\ {}_1) - f(\ {}_2) + a\left(f(\ {}_2) - f(\ {}_1)\right) = (a-1)\left(f(\ {}_2) - f(\ {}_1)\right)$$

$f(x) \qquad {}_1 \le \ {}_2, f(\ {}_2) - f(\ {}_1) \le 0 \quad a - 1 \le 0$

$$\therefore F(x) \ge 0 \Rightarrow \frac{1}{a}\int_0^a f(x)dx \ge \int_0^1 f(x)dx$$